THE STORY OF Q

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IF SCIENTISTS were as fair as Humpty Dumpty in paying words extra for overtime work, the ubiquitous Q would come near the top of the payroll. It all started with K. S. Johnson.* Many scientists have contributed new words to our vocabulary. To only one, however, has come the distinction of elevating a letter of the alphabet into a word of everyday use in many and diverse fields. Little did Johnson dream, when he first used the symbol Q to represent the ratio of reactance to effective resistance in a coil or a condenser, that within a span of some 30 years this same symbol would be commonly used to describe an attribute of such dissimilar things as a resonant circuit, a spectral line, a mechancial vibrator, and a bouncing ball. The story of this expanding usage of the 17th letter of the alphabet makes an interesting study for the scientific etymologist.

Coils and Condensers

The tale begins in the teens of this century. It was then the usual practice, in appraising the quality of the devices which were then known as coils, and which engineers have now become educated to call inductors, to use the ratio of effective resistance to reactance as a sort of figure of merit. Because it was related to dissipation, this ratio was often designated d, and in fact it is now commonly referred to as the dissipation factor. Strictly speaking, d is not a figure of merit but a figure of demerit, since the normally desirable condition of minimum losses occurs as the value of d moves towards zero.

As early as 1914, Johnson came to realize that a ratio of greater utility for many purposes than the one in vogue was its reciprocal. Johnson was aware that the ratio d is convenient for certain mathematical computations, since it permits the combining of different sources of loss by direct addition. He observed, however, that in practical cases d would usually involve one or more zeros preceding the significant figures, whereas the reciprocal could usually be taken as a whole number. The same sort of logic, therefore, which leads to the common use of impedance, and avoidance of admittance, argued for putting reactance in the numerator of the ratio.

For a time Johnson designated the ratio of reactance to effective resistance of a coil by the symbol K [1]. It was in 1920, while working on the practical application of the wave filter which G. A. Campbell had invented some years before, that he for the first time employed the sym-

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bol Q for his parameter [2]. His reason for choosing Q was quite simple. He says that it did not stand for "quality factor" or anything else, but since the other letters of the alphabet had already been pre-empted for other purposes, Q was all he had left.

Initially Johnson used a capital Q for coils, and a small q for the corresponding ratio in condensers (now renamed capacitors). Before long, however, he began to apply capital Q to both coils and condensers, using subscripts where differentiation was needed. The first printed use of Q seems to be in Johnson's U.S. patent No. 1,628,983, where it is applied to the coils in an electrical network. In Johnson's classic treatise on "Transmission Circuits for Telephonic Communication" [3] the symbol Q appears in a number of places to designate the parameter which he called the "coil dissipation constant." Subsequently this was shortened to "dissipation constant," applying to both coils and condensers [4]. The terms "coil constant" and "condenser constant" also were used to some extent. Later on, V. E. Legg coined the apt name of "quality factor," while others tried to introduce such terms as "storage factor" and "figure of merit." But none of these appellations could prevail over the terse and trenchant Q.

Another measure which has frequently been used for a reactive element is the power factor, i.e., the ratio of active power to total volt-amperes. At any two terminals the power factor is the cosine of the phase angle of the impedance, whereas Q is the tangent of the phase angle, neglecting sign. Thus for the common case where reactance is large compared with resistance, the power factor is substantially equal to the dissipation factor d. Power engineers, who are accustomed to using power factor to designate the ratio of active power to total volt-amperes, might occasionally, if they experience a need for a ratio greater than unity, find it advantageous to borrow Q from the communication engineer.

Others before Johnson had made use of the ratio of reactance to resistance for either an inductor or a capacitor (to use modern parlance). Johnson's role was to popularize this ratio and to assign to it the contagious symbol Q. He did not intend to apply Q to anything except the ratio of reactance to resistance, whether of an inductor, a capacitor, or any two-terminal network. In fact, he was somewhat disturbed, as originators of terminology often are, when others began to extend his usage—an extension which has gone so far that a few modernists would even like to ban Johnson's original meaning.

Resonant Circuits

What happened next? First was the discovery that Q was a convenient symbol to apply to a resonant circuit. It was noted that the high-frequency losses in a well-constructed capacitor were ordinarily negligible in comparison with those of an inductor. Hence the high-frequency resistance of the usual inductor and capacitor resonant circuit could be assumed equal to the inductor resistance, and the Q of the resonant circuit could therefore be assumed the same as the Q of the inductor. In those cases where the capacitor resistance could not be neglected, the Q of the resonant circuit was $Q_L Q_C/(Q_L+Q_C)$, where Q_L and Q_C are the Q's of the inductor and capacitor, respectively, at the resonant frequency. It is noteworthy that this use of Q for a resonant network is uniquely related to the resonant frequency, whereas Q when applied to an impedance is a property at any specified frequency.

At this point it became apparent that Q as applied to a resonant circuit was an already recognized parameter which for want of a better name had previously been called "sharpness of resonance" [5]. This permitted the establishment of several relationships which today are elementary. Curves like those of Figure 1 could be drawn to show current *versus* frequency as a function of Q for a series resonant circuit, and analogous curves for the impedance of a parallel resonant circuit.

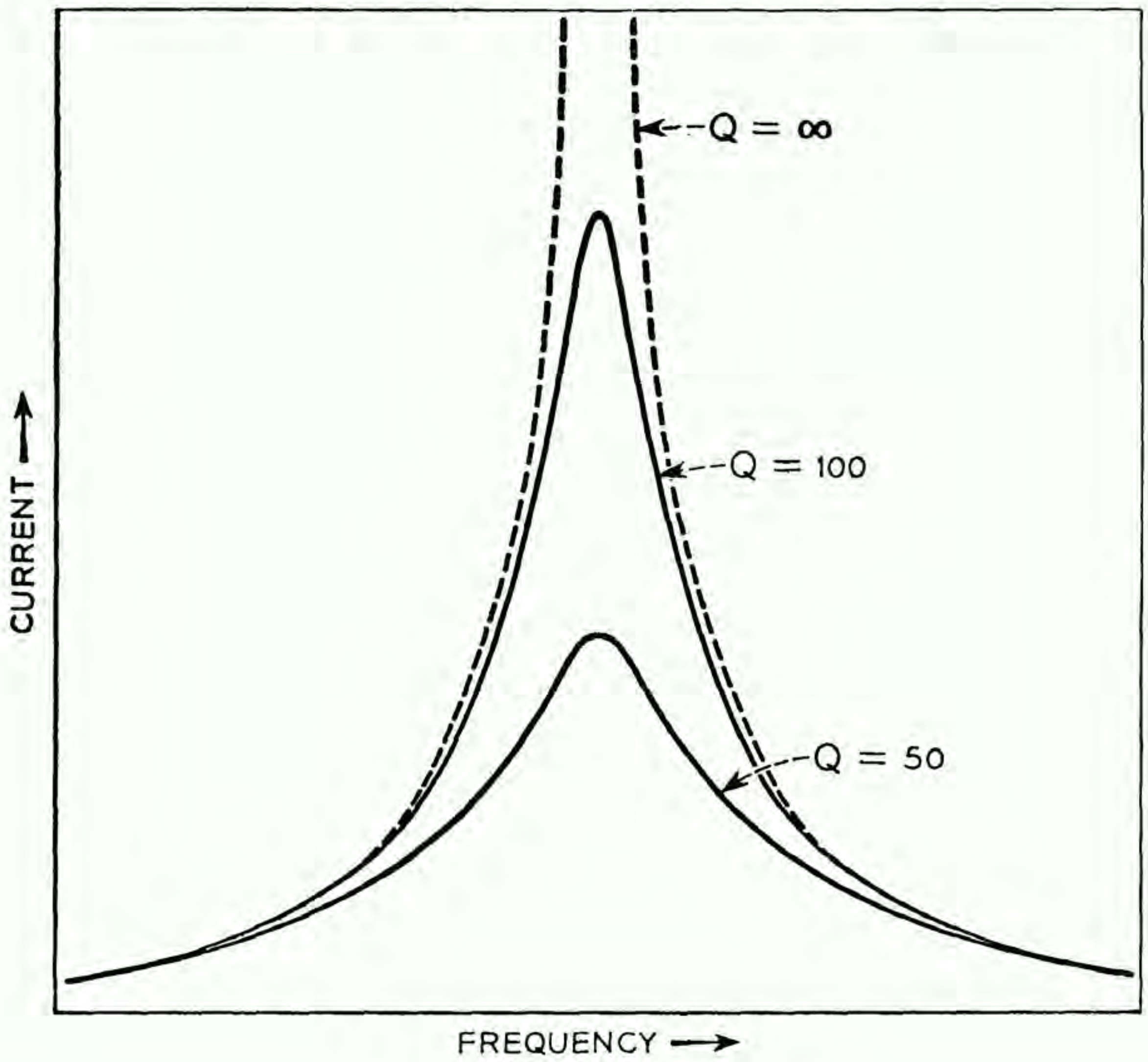

FIG. 1. Series resonance.

Once identified with sharpness of resonance, Q was seen to bear a close relationship to a familiar parameter of an oscillatory wave train of continuously decreasing amplitude. This parameter was the logarithmic decrement, which was defined as the natural logarithm of the ratio of two successive maxima in a damped wave train. Thus in Figure 2 the logarithmic decrement δ is equal to $\log_e (AB/CD)$.

Solution of the differential equation for a resonant circuit comprising resistance, inductance, and capacitance in series gives for the logarithmic decrement δ the value $\pi R/\omega L$, or $\pi R\omega C$. Hence Q equals π/δ. While much importance attached to logarithmic decrement in the earlier days of radio, in connection with the damped waves produced in a spark transmitter by the sudden discharge of a condenser through a spark gap, Q was so much better adapted to continuous wave techniques that today the term logarithmic decrement is all but forgotten.

Many relations previously established for the logarithmic decrement were restated in terms of Q. Thus the number of complete oscillations necessary to reach a given ratio ρ of initial amplitude to final amplitude is Q/π times $\log_e \rho$. From this we learn that for a Q of 100, for example, the number of oscillations necessary to reach one per cent of the initial value is 146, while for a Q of 200 twice as many oscillations would be required.

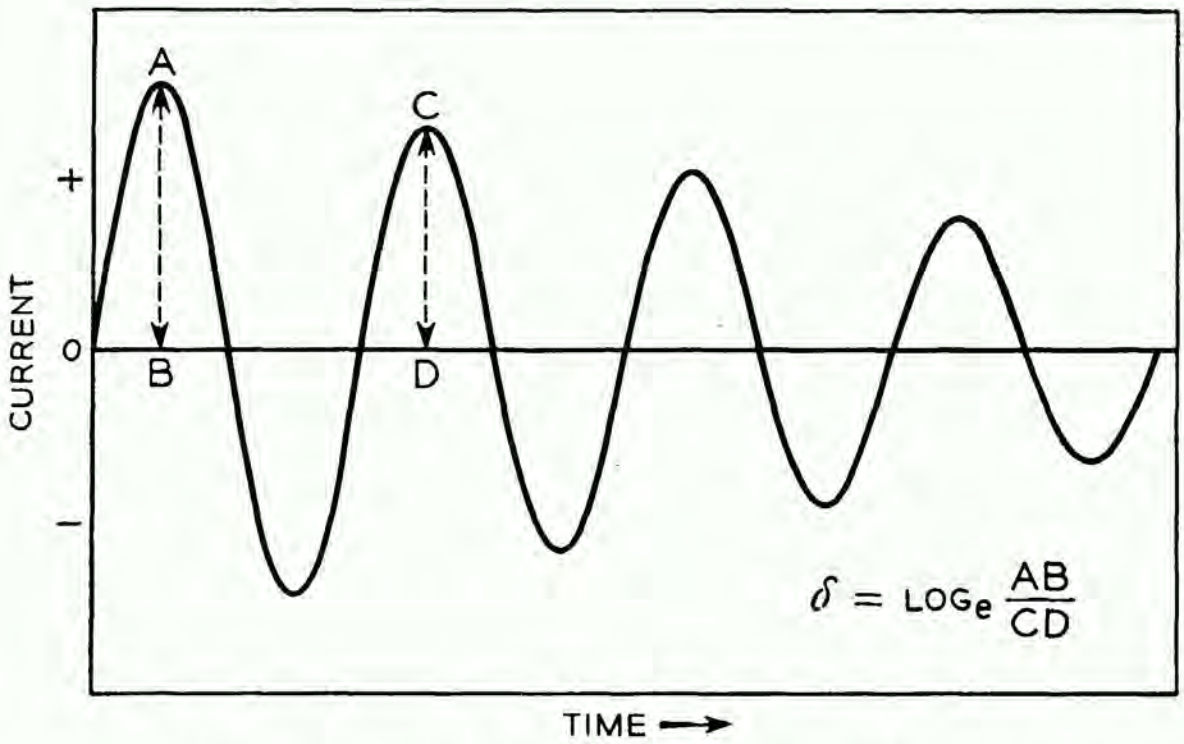

FIG. 2. Logarithmic decrement for damped wave-train.

Through pursuit of the logarithmic decrement relationship, simple algebraic manipulations yielded a useful physical picture, namely, that for a simple resonant circuit the ratio of the maximum energy stored in either the coil or the condenser to the energy dissipated per cycle is equal to $Q/2\pi$. Also, for larger values of Q, the voltage across either the inductor or the capacitor of a series resonant circuit is substantially equal to Q times the applied voltage [6]. Similarly the current through either the inductor or the capacitor of a parallel resonant circuit is equal to Q times the total current.

An even more interesting relationship was found between Q and the shape of the resonance curve [7]. Through derivations too detailed for inclusion here, it turns out that for a curve showing magnitude of impedance or admittance of a resonant circuit *versus* frequency, Q is approximately equal to the ratio of the resonant frequency to the width of the resonance curve between the points, on either side of resonance, where the ordinate is, respectively, $1/\sqrt{2}$ times the maximum or $\sqrt{2}$ times the minimum ordinate.

Resonant Devices

The groundwork was now complete, and anyone could take off in any direction. A natural extension from the inductor and capacitor resonant circuit was to apply Q to any resonant structure or device. For this purpose the definition of Q in terms of energy storage and dissipation was directly applicable, while the relation for the shape of the resonance curve was broadened by stating it in terms of response. Thus Q became equal to the ratio of the resonant frequency to the bandwidth between those frequencies on opposite sides of resonance (known as "half-power points") where the response of the resonant structure differs by 3 db from that at resonance. The use of Q with such connotations for tuning forks, piezoelectric resonators, magnetostrictive rods, and the like soon became commonplace.

Resonant Transmission Lines

Resonant transmission lines came next. The standing wave patterns for open-circuited or short-circuited lines, exhibiting maxima and minima at "resonance" points located at quarter-wave multiples from the terminating end, were, of course, familiar from classical derivations. The trend to higher frequencies, especially for radio communciation, made it increasingly advantageous to utilize such resonant-line phenomena for oscillator frequency control, voltage step-up, impedance inversion, and the like. Since the curve of line impedance in the vicinity of resonance is essentially similar to that of a resonant circuit, it was a natural step to apply the factor Q to a resonant transmission line. F. E. Terman [8] showed that the Q of such a line is equal to $\pi f/\alpha V$, where f is the resonant frequency, α is the real part of the propagation constant, and V is the group velocity, i.e., the velocity with which signals are transmitted.

Cavity Resonators

At frequencies upward from about 1000 mc (commonly referred to as microwaves) resonant transmission lines usually give way to cavity resonators. The cavity may be cylindrical, parallelepipedal, spherical, or some other shape, depending on end use. Regardless of shape, a cavity resonator has an infinity of resonant frequencies, starting at a minimum value and becoming more closely spaced with increasing frequency. Each resonance corresponds to a particular standing wave pattern of the electromagnetic field, which is called a resonant mode, and for which the cavity may be considered as a single tuned circuit (with L and C not defined). The Q of a cavity resonator for any mode is therefore definable in terms of losses or bandwidth, and turns out to be a function of the ratio of internal volume to internal area. A general expression for the Q

of a cylindrical cavity resonator was derived by S. A. Schelkunoff in 1934 in unpublished lecture course material. Values of Q for different shapes of cavity resonator and different modes were published in 1938 by W. W. Hansen [9]. The practical designer must distinguish between (*a*) the nonloaded or basic Q resulting from theoretical consideration of the cavity without external coupling, and (*b*) the loaded or working Q obtained when microwaves are excited within the cavity by one or more orifices, loops, or probes.

Material Q

While these various excursions were under way, engineers concerned with magnetic or dielectric materials discovered that Q was a convenient device for expressing the dissipation properties of a material, as distinct from other sources of loss in the device in which the material is used. For this purpose Q may be defined in terms of energy storage, that is, Q equals 2π times the ratio of maximum stored energy to the energy dissipated in the material per cycle.

Spectral Lines

In the search for more precise standards of frequency and time, scientists in recent years have turned to a new tool—atomic and molecular processes. The quantum theory, first advanced by Planck, states in substance that changes of energy in atoms and molecules are not continuous but occur in steps, each step being the emission or absorption of an amount of energy, called a quantum, which is equal to the product of Planck's constant and the radiation frequency. The frequencies which correspond to such transitions between characteristic energy states for different atomic and molecular structures are referred to as spectral lines. While many of the transitions in atoms and molecules involve energies which correspond to very high frequencies, there are a number of low energy transitions corresponding to frequencies in the microwave region.

One method of employing these phenomena for obtaining frequency standards is to expose a gas to electromagnetic fields in sharply defined frequency bands where low energy transitions are produced within the molecules [10]. Ammonia gas (NH_3), for example, exhibits a strong absorption band at 23,870 mc. Another method of using these fundamental properties of matter is to employ microwave fields of sharply defined frequency to deflect an atomic or molecular beam [11]. In either case the shape of the curve of response *versus* frequency is very similar to that of a resonant circuit, which makes it convenient to describe the selectivity by an equivalent Q defined as the ratio of the frequency of maximum response to the 3-db bandwidth.

Definitions

So the irrepressible symbol has ranged here, there, and everywhere. For years no one tried to brand the maverick. Usage was expanding so fast that people shied away from exact definition. When the ASA C42 Standard Definitions of Electrical Terms were published in 1941, Q was omitted because it was looked upon as a symbol without a name. It was not until a revision of those definitions was gotten under way in 1947 that a committee attempted to write a complete definition of Q. Even then, while the basic concepts were clear enough, precise wording proved difficult, and will doubtless need modification as usage spreads and changes.

Values of Q—Reactors and Resonators

While it has been implied that Q is frequently large compared to unity, it is now time to have a closer look at its size. Values which are intended to indicate order of magnitude, and not for engineering design, are discussed below, and the general range of values is illustrated in Figure 3.

To start, as K. S. Johnson did, with the inductor, good present-day accomplishment for air or molybdenum permalloy powder cores at moderately high frequencies is represented by Q's in the range from 50 to 250. The introduction of the ferrites, whose high resistance practically eliminates high frequency eddy current losses, has opened up new vistas, and permitted economies in volume, weight, and cost. Q's of 300 to 500 in the frequency range 50–100 kc are now realized commercially in filter inductors of shell-type ferrite construction having an over-all volume of slightly more than a cubic inch. A rather spectacular recent achievement was the construction of an experimental ferrite inductor with a Q of more than 1000.

The Q of capacitors covers a wide range. Electrolytics give 15 or 20 at 1 kc and fall off rapidly at higher frequencies. Ceramic types have Q's less than 100, oil impregnated paper capacitors values of several hundred, and mica and air capacitors values in the thousands or tens of thousands.

For real mammoth Q's, engineers turn to quartz crystals (Fig. 4). Units of the air-enclosed type, as produced commercially for either frequency control or network applications afford Q's from 10,000 to 100, 000. In precision-type crystal units, particularly those designed for use in frequency standards at 100 kc or 5 mc, much higher values of Q are obtained by selection of the raw material, by careful preparation of the surface of the plate, by design of the supporting system to prevent energy absorption, and by evacuating the enclosure in order to reduce radiation to the atmosphere. By such means, Q's of one to two million are realized in commercial production and four to ten million in development models. The Q's obtained in quartz crystal units are the highest Q's yet achieved in passive man-made devices. Of course, by using ampli-

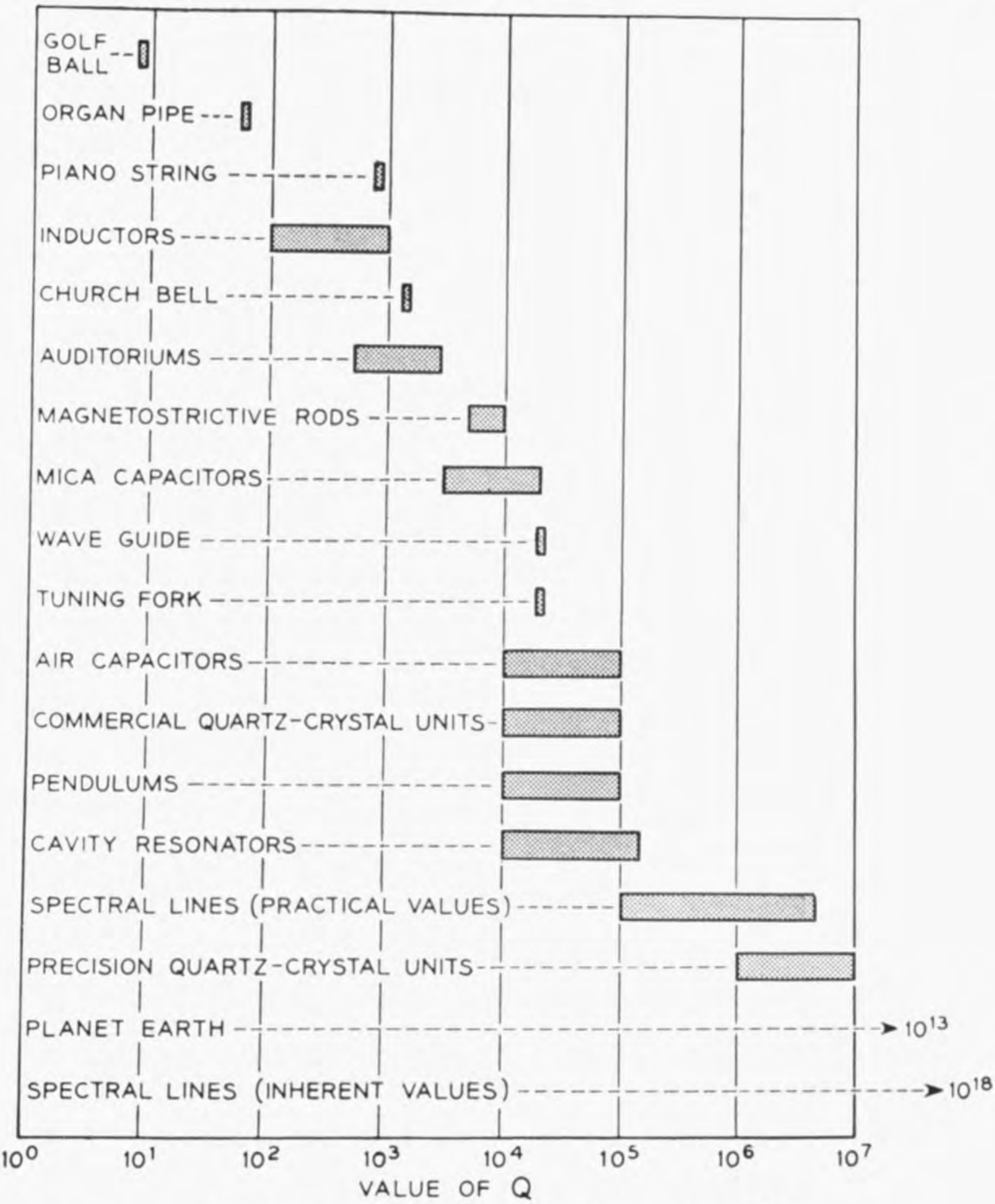

FIG. 3. Q's for various phenomena and devices.

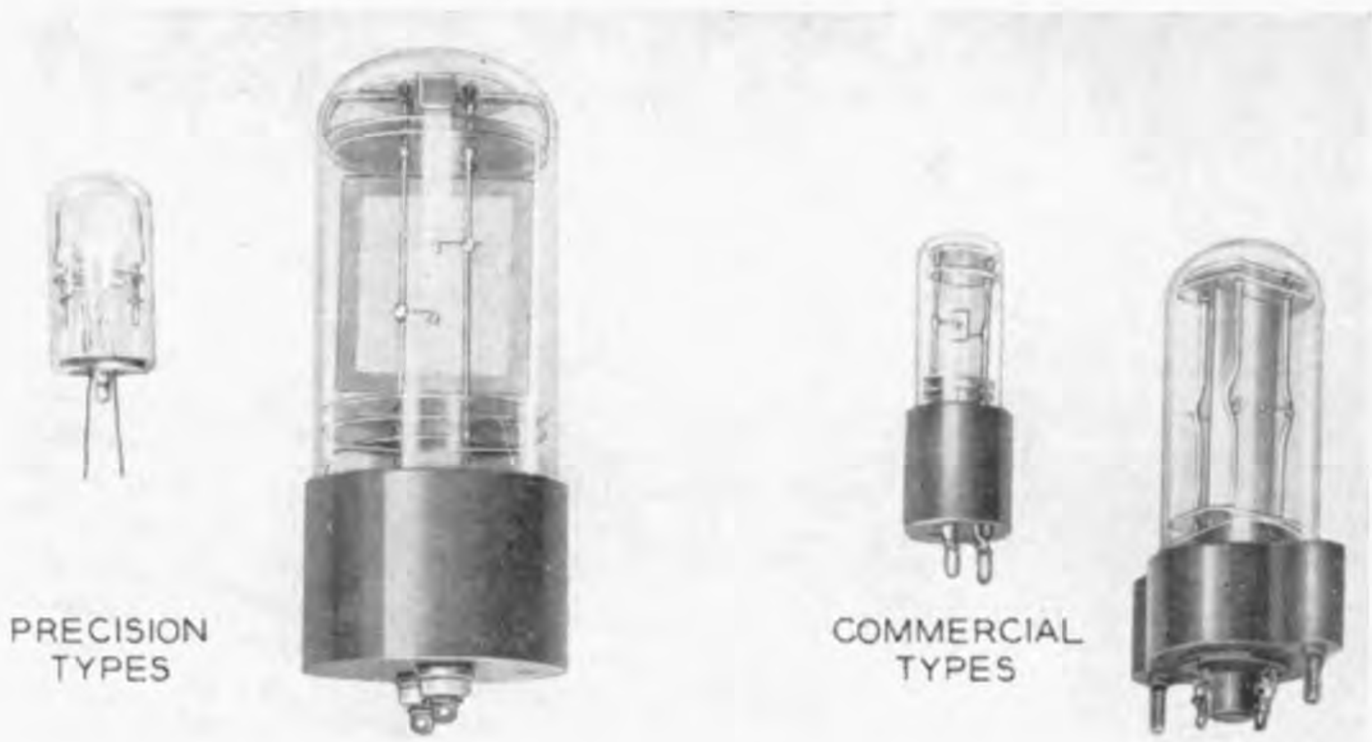

FIG. 4. Quartz crystal resonators.

fications to offset losses, Q can in effect be made infinite, but this is an area outside present consideration.

In cavity resonators employed as frequency-fixing elements, as frequency standards, as adjustable frequency meters or wavemeters, or as selective elements in filters, amplifiers, etc., a moderate value of Q, from a few hundred to perhaps 10,000, is usually sufficient. In order to avoid confusion with other modes, the dominant or fundamental mode, i.e., the mode with the lowest cutoff frequency, is usually selected for such applications. Another important use of cavity resonators is in so-called "echo boxes" (Fig. 5) for determining the over-all performance of a radar

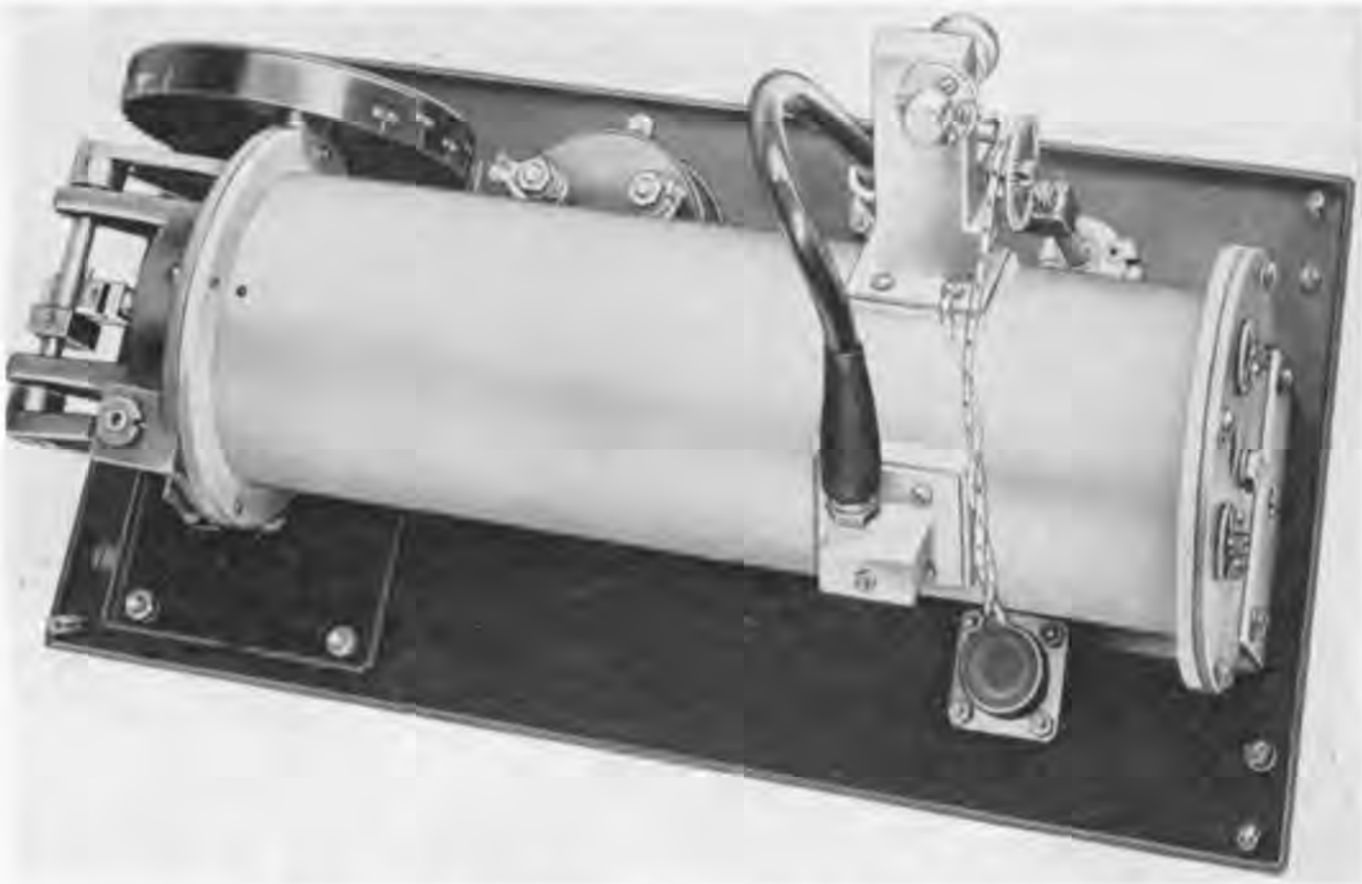

FIG. 5. Cavity resonator for radar testing.

by coupling the radar output into the cavity and observing the time interval required for the signal returned to the radar receiver to disappear into the noise [12], [13]. This type of application requires extremely high values of Q, ranging from 40,000 to 200,000 for frequencies from 1000 to 25,000 mc. Cylindrical resonators are usually employed, frequently at a mode above the fundamental.

The Q's obtained in other resonant systems and devices vary over quite a range. Magnetostrictive rods and tuning forks give fairly high Q's, while musical devices such as piano and violin strings, organ pipes, church bells, etc. have rather low values.

In a majority of situations the desired value of Q is the highest one economically attainable. In fact, the label "high Q" is often interpreted to mean "high quality." Sometimes, however, a minimum Q is the desideratum. For example, circuit designers who are plagued by the high-frequency reactance of resistors find it comforting to obtain a Q of about 0.001 in a deposited carbon resistor at 100 kc. Cases also arise, however, where it is desired neither to maximize nor to minimize Q. One such is a reverberant room or hall, which has a Q derivable from the curve of decay of sound intensity or from the reverberation time, i.e., the time for the sound intensity to drop 60 db. By suitable sound absorbing techniques, the Q can be reduced to a very low value, making a "dead" room. On the other hand, high reflectivity for all interior surfaces gives too live a room. In general there is an optimal Q which lies somewhere in between, and depends on a number of factors, including the volume of the room and the type of sound for which it is utilized. Q's of about 250 to 350 provide good room acoustics for moderate sized halls, while larger values are preferable for large auditoriums and cathedrals.

Bouncing Balls

As suggested by R. S. Duncan, the concept of Q may be extended to a bouncing ball, the Q being determined by measuring the height of successive rebounds when the ball is dropped on an unyielding surface. Theoretically the test should be made in a vacuum but the results in air are about the same. For moderate heights of drop, golf balls give Q's of about 8 or 9, tennis balls slightly less. In the persistent search for new ways to attract consumer attention, it may be that the manufacturers of golf or tennis balls will some day get around to advertising a high-Q product. Should they become overboastful, however, it could be pointed out that if a golf ball had the two million Q of a precision crystal, it would after 440,000 bounces still be rebounding to 50% of the original height.

Rotating Bodies

Having gone so far with Q, it is perhaps not too much of a stretch to apply it to a rotating body subject to frictional deceleration. The ratio of the stored energy to the energy lost per cycle suffices to determine the Q. Thus the gyroscope of a gyrocompass might be considered to have a Q in the order of a million or so.

Traditionally nothing is as constant as the earth's rotation. Our standard of time is the mean solar day and our standard of frequency derives from this. In reality, however, these standards are measurably unstable. The earth is constantly slowing down, mostly because of tidal friction, and in addition there are irregular variations in rotational rate whose cause is not wholly understood. Observations of the motions of a number of heavenly bodies indicate that the length of the day is increasing at the rate of 0.00164 second per century [14]. Neglecting irregular variations, therefore, we can as a matter of academic interest figure out the Q of the earth as a rotating body subject to a retardation which may be assumed constant over the present span of human concern. As might be expected, the value of Q thus determined turns out to be very large—

about 10^{13}, which is far beyond the range of Q's obtained in man-made devices.

Atomic and Molecular Q's

The Q which is inherent in molecular transitions is very large, 10^{15} to 10^{18}. Considerable broadening is caused, however, by molecular collisions and by Doppler effect, so that under working conditions the Q obtained by the absorption method may be of the order of 100,000 to one million [15]. With the beam deflection method, using cesium or thallium atoms, it appears possible to obtain Q's of 30 million to 50 million. These quantum-mechanical phenomena (either absorption or beam deflection) seem to offer the best promise for non-aging frequency and time standards which will eventually be needed to supplement the changing standard provided by the earth's rotation. In the meanwhile, Q, having already ranged from the atom to the planet, will doubtless move on to subatomic and cosmic realms.

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