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A *Q*-rious Tale; the Origin of the Parameter *Q* in Electromagnetism

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SUMMARY

The history of the use of Q is traced in fields other than geophysics.

1 INTRODUCTION

One definition of the parameter Q often used to characterize a damped oscillatory process is that Q^{-1} is $(\frac{1}{2}\pi)$ times the fractional energy loss per cycle. In some geophysical contexts it appears that it may be more appropriate to use a quantity proportional to the reciprocal of Q. A list of the various possibilities is given by Anderson (1967, p. 137).

2 THE HISTORY IN ELECTROMAGNETISM

I set out to explore the history of Q in other fields and if possible to pursue it to its beginning and this has proved unexpectedly difficult. Dr Anderson suggests that Q came into geophysics in the 1950s and this agrees with the conjecture that it came by way of the theory of waveguides and microwave resonators, widely studied in the war of the 1940s. However I think it is clear that it originated in 'lumped' circuit theory. Although some authors of textbooks disclaimed knowledge of its origin this seemed the most hopeful line of research and Dr J.A.Ratcliffe joined enthusiastically in the hunt. Q is on p. 14 of the 9th edition (1952) of his *Physical Principles of Wireless* but not in the first edition (1929).

To summarize briefly the notation used, we have that for a (LCR) circuit with small damping the current is proportional to $\exp(-Rt/2L + i\omega_0 t)$ where $\omega_0^2 = 1/LC$. The logarithmic decrement Δ and quality factor Q are here defined by

$$Q = \pi/\Delta = (L/C)^{\frac{1}{2}}/R = \omega_0 L/R.$$

(Other definitions appear in the literature; one has to cultivate a robust attitude to the intrusion of 2 or π.) Under a forced EMF proportional to $\exp(i\,\omega\,t)$ the difference between the two frequencies at which the power is half that at resonance is ω_0/Q.

Dr Ratcliffe points out that

> Before the early 1920s transmitters used spark excitation of a circuit, which provided a decaying train of oscillations. Theory concentrated on the decay of these oscillations, measured by *log dec*, but after the start of commercial broadcasting the transmitter circuits were driven by valves and radiated 'continuous waves' on a single frequency. The receiving circuits had to be accurately tuned to this frequency and theory turned its attention to the 'sharpness of resonance' of the circuit, measured by Q.

A chance remark of Professor G.A.Barnard led me to the *Admiralty Handbook of Wireless Telegraphy*. In the 1939 edition Vol. 1, chapter V, para. 302 is headed *'Q' and Selectivity*, and Vol. 2, Section D40 is devoted to it. The 1931 edition has no Q in the index but in para. 254,

> The square root of L/CR^2 *viz.* $(L/C)^{1/2}/R$ is taken as a criterion of *selectivity* (para. 492) of a series resonant circuit i.e. its response at frequencies away from resonance compared to its response at the resonant frequency. It will be seen later (para. 397) that $(L/C)^{1/2}/R$ is in inverse proportion to the expression derived for the logarithmic decrement of an oscillatory circuit in which a free oscillation has been set up.

Palmer (1928) defines selectivity S as $2(L/C)^{1/2}/(\pi R)$ and refers to Circular 74, p. 37 (1918) of the American Bureau of Standards where $(L/C)^{1/2}/R$ is used as the criterion of sharpness of resonance, but it is not called Q.

Where the name Q was *first* adopted remains a mystery. What does emerge is that it was adopted for the discussion of resonance, whereas its reciprocal is more appropriate to the discussion of damping of a free oscillation. To one who is not a geophysicist it seems strange that Q is used in contexts where nearly all the formulae contain its reciprocal.

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