Long-Periodical Variations of Earth Rotation, Determined from Reconstructed Millennial-Scale Glacial Sea Level

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Abstract
The variations of the Earth rotation in time are caused by the gravitational influence of Moon, Sun and planets, displacements of matter on the Earth surface, inside the Earth liquid core and core-mantle boundary. An important part of Earth rotation excitation is the influence of the Mean Sea Level (MSL) changes, due to polar ice sheets variations, followed by changes of the axial momentum of inertia. Significant polar ice variations occur during the last glacial-deglacial cycles. Recently the glacial sea level variations have been reconstructed for the last 380Kyr by Siddall et al. (2003). These data are used to determine the long-periodical variations of Universal Time UT1 and Length of Day (LOD). The long-periodical components of MSL and Earth rotation are determined by means of spectral analysis, Fourier approximation based on the Least-Squares estimation of trigonometrically coefficients and autoregressive time series analysis (ARIST) of the unknown frequencies. The estimated periodicities of MSL and UT1 variations are compared with the variations of Earth precession and Earth orbit parameters. The results will expand our knowledge about the global Earth processes and mutual influences between long-term climatic variations, hydrological cycles, Earth Orientation Parameters (EOP) and Earth orbit at millennial time scale.

Keywords: mean sea level, Length of Day LOD, Universal Time UT1

Introduction

Recently the Mean Sea Level (MSL) variations have been reconstructed for the last 380Kyr by Siddall et al. (2003). The data consist of 3 series with non-evenly distributed points. The longest series is shown in Fig.1. These data are evenly interpolated with step of 500a. The glaciations cause significant MSL variations, water redistribution between the ocean and polar ice, and change of the Earth axial moment of inertia, which is connected with long periodical variations of the Earth rotation, expressed as Length of Day (LOD) and Universal Time (UT1) oscillations. It is possible to determine the long-term glacial variations of LOD and UT1 by means of the MSL data.

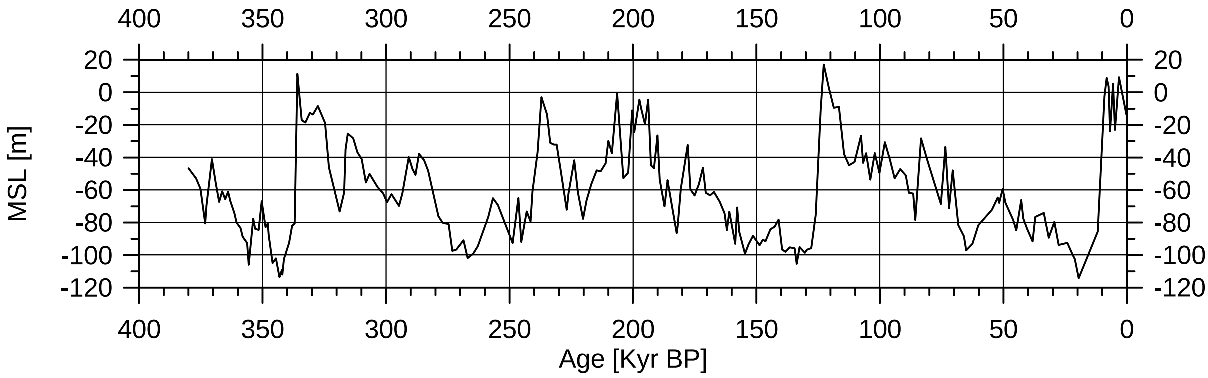

Figure 1. Reconstructed mean sea level for the last 380Kyr by Siddall et al. (2003).

The Length of Day is the difference between the astronomically determined duration of the day and 86400 SI seconds. The relationship of the angular velocity of the Earth with LOD is (http://www.iers.org/IERS/EN/Science/EarthRotation/UT1LOD.html)

$$\omega = \omega_0 - \omega_1 L_{OD},$$

(1) $\omega_0 = 72921151.467064,$

$\omega_1 = 0.843994809,$

where ω is the Earth angular velocity, L_{OD} – the Length of Day in milliseconds, ω, ω_0 and ω_1, are in picoradians/s. According (1), the variations of LOD depend on the variations of the Earth angular velocity by the formula

$$L_{OD} = -\frac{\Delta\omega}{\omega_1} \tag{2}$$

Interconnection between MSL and LOD variations

Let consider a homogeneous sphere with radius R=6371km, density 5.519g/cm^3 (Neff and Zitewitz, 1995), inertial moment C, mean angular velocity ω_0 and constant mass M. From the conservation of angular momentum, the small changes of the inertial moment ΔC, due to variations of the radius ΔR, are connected with a corresponding change of angular velocity $\Delta\omega$ by the expressions

$$\begin{aligned} &\Delta C\omega_0 + C\Delta\omega = 0, \\ &C + \Delta C = \frac{2}{5}M\left(R^2 + 2R\Delta R + \Delta R^2\right). \end{aligned} \tag{3}$$

The relationship of the variations of the radius and angular velocity is

$$\Delta\omega = -\frac{2\Delta R\omega_0}{R}. \tag{4}$$

From (2) and (4) the relationship of L_{OD} and ΔR is

$$\Delta L_{OD} = \frac{2\Delta R\omega_0}{\omega_1 R}. \tag{5}$$

The transfer from homogeneous elastic sphere to the real Earth need to replace the variations of radius ΔR by equivalent changes of the mean sea level ΔM_{SL}. It is necessary to determine transfer coefficient k between the homogeneous elastic sphere and the real Earth, which value represent the ratio between ΔM_{SL} and ΔR

$$k = \frac{\Delta M_{SL}}{\Delta R}. \tag{6}$$

Involving coefficient q of ice sheets influence on the moment of inertia, we obtain from (5) and (6)

$$L_{OD} = \frac{2\omega_0(1-q)}{k\omega_1 R}\Delta M_{SL}, \tag{7}$$

The value of the coefficient k depends on the mean density of sea water at the ocean surface, total ocean surface and the moment of inertia of the thin ellipsoidal shell over the ocean with thickness equal to ΔM_{SL}. In the case of small MSL variations (significantly less than 1m), the water lost occur from all Earth surface, more intensive from the free water surfaces and less intensive from the ground. The coefficient k is approximately equal to 6 in this case (Chapanov and Gambis, 2010). The MSL changes are more than 100m during the glaciations, so the level of the water lost from the ground is neglectful and the coefficient k is

$$k = \frac{D_E S_E I_{ES}}{D_O S_O I_{OS}}, \tag{8}$$

where D_E is the mean Earth density, D_O – the mean sea water density at the ocean surface, S_E – the total Earth surface, S_O – the global ocean surface, I_{ES} - the moment of inertia of the thin ellipsoidal shell over the Earth, I_{OS} - the moment of inertia of the thin ellipsoidal shell over the ocean. The mean sea surface water density is 1.025g/cm^3, or the ratio D_E/D_O is equal to 5.38. The total Earth surface is 510×10^6 km^2, the global ocean surface - 361×10^6 km^2, and their ratio – 1.414. The proper calculation of the LOD glacial variations need also knowledge of the surfaces covered by ice during the maximal glaciations and their axial moments of inertia. The MSL decrease leads to decreasing of the Earth axial moment of inertia, while the redistributed water over the ice sheets and corresponding increase of the ice thickness leads to some increase of the axial moment of inertia. The relative effect of the ice sheets on the I_{OS} is

(9) $q = \dfrac{S_O I_{Ic}}{S_{Ice} I_{OS}}$,

where S_{Ice} is the total ice surface and I_{Ic} - the total moment of inertia of thin ellipsoidal shell over the continental ice sheets. The sea ice does not affect the moment of inertia changes, due to its hydrostatical equilibrium with the ocean water.
Let the surface glacial data is represented over a grid with size $\alpha\times\alpha$ degrees and the geocentric coordinates of the center of the n-th grid element are geocentric distance r_n, longitude λ_n and latitude θ_n. If a and b denote Earth equatorial and polar radii (a=6478.137km, b=6356.572km), then the geocentric distance r_n is

(10) $r_n = \sqrt{\dfrac{a^2 b^2}{a^2 \sin^2\theta_n + b^2 \cos^2\theta_n}}$.

The distance h_n from the center of the n-th grid element to the Earth axis of rotation is

(11) $h_n = r_n \cos\theta_n$.

The surface s_n of the n-th grid element is approximately

(12) $s_n = 4\pi^2 \left(\dfrac{\alpha}{360}\right)^2 r_n h_n$.

The surface S of a given Earth area is the sum of its grid elements surfaces

(13) $S = \sum_n s_n$.

The axial moment of inertia I of thin ellipsoidal shell with unit density and thickness over this area is

(14) $I = \sum_n s_n h_n^2$.

The axial moment of inertia of ellipsoidal shell with unit density and thickness over the Earth is 1.38×10^{14} km^4 and over the ocean - 1.0×10^{14} km^4. Their ratio is 1.38, and according (8) the value of the transfer coefficient k between the homogeneous elastic sphere and the real Earth is k=10.5.
The scientific publications concerning last glaciations contain rather enough information about the continental and the ice sheets distribution. Nowadays it is well known that the continental ice sheets during the last glacial maximum reach latitudes between 40° and 50° in the North America and between 50° and 60° in Euro-Asia. Roughly, this area is almost circular with radius 4400km (corresponding to 40° over the meridian). The center of this area is shifted by 10° from the North Pole to the Greenland direction. The South Pole ice sheet cover almost circular area with latitude above 50° (Paul and Schäfer-Neth, 2003; Schäfer-Neth and Paul, 2003), thus its radius is approximately the same as the North Pole ice sheet.

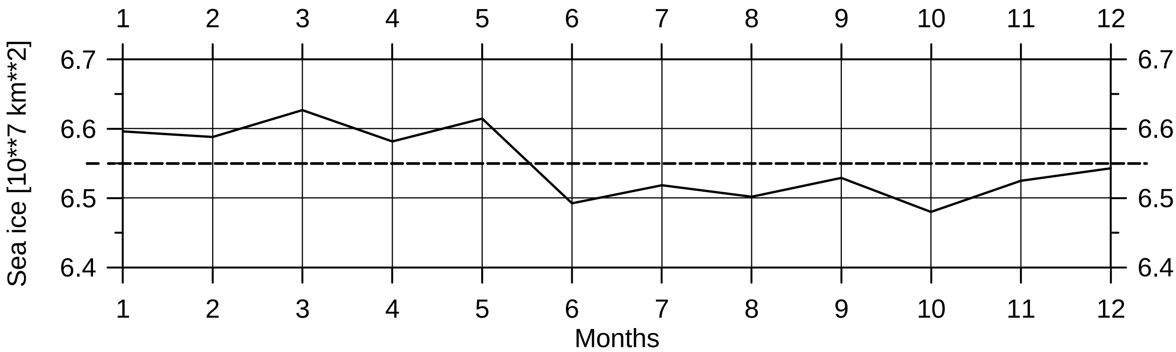

Figure 2. Monthly sea ice extent variations during the last glacial maximum according Schäfer-Neth and Paul (2003). The mean sea ice surface is 65.5×10^6 km^2.

Reconstructed data of gridded global sea surface temperature and salinity during the last glacial maximum is available in IGBP PAGES/World Data Center for Paleoclimatology, Boulder (Schäfer-

Neth, C. and A. Paul, 2003, Gridded Global LGM SST and Salinity Reconstruction, Data Contribution Series #2003-046, NOAA/NGDC Paleoclimatology Program, Boulder CO, USA). It is possible to compute the surface and the axial moment of inertia of thin ellipsoidal shell over the ocean and areas with sea ice by means of these data and formulae (10-14). The mean total sea ice surface is 65.5×10^6 km^2 (Fig.2) with mean axial moment of inertia of the thin ellipsoidal shell over the sea ice 5.09×10^{14} km^4 (Fig.3).The total circular ice surface over polar caps is 121.8×10^6 km^2 and after subtracting the mean total sea ice surface, we obtain the continental polar surface, covered by ice - 56.3×10^6 km^2. Taking into account the surface of Himalaya ice (approximately 4×10^6 km^2), we determine the total continental surface of about 60×10^6 km^2, covered by ice during the last glacial maximum and the total ice surface (the sum of continental and sea ice surfaces) – approximately 126×10^6 km^2.

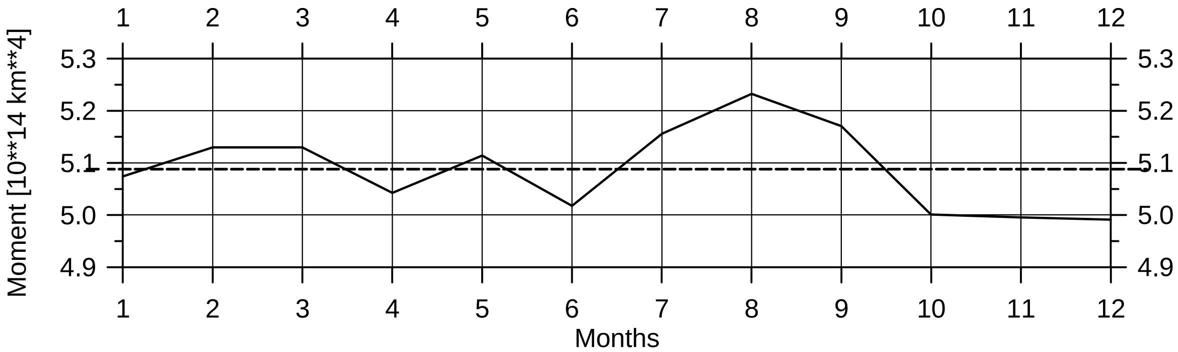

Figure 3. Monthly values of the axial moment of inertia of thin ellipsoidal shell over the sea ice during the last glacial maximum according Schäfer-Neth and Paul (2003). The mean value of the moment of inertia is 5.09×10^{14} km^4.

Assuming even distribution of the ice, we determine that the mean ice thickness during the last glacial maximum is proportional to the MSL variations and the ratio between the total ocean surface S_O and total ice surface S_{Ice}, or 2.87 when the mean sea level was –114m. The modern value of the above ratio is 12.45 (when ΔM_{SL}=0m). According these two values, the linear dependence between the ratio S_O/S_{Ice} and ΔM_{SL}, expressed in meters, is

$$(15)\quad \frac{S_O}{S_{Ice}} = 0.084\Delta M_{SL} + 12.45\,.$$

The moment of inertia of thin ellipsoidal shell over the South polar ice is 5.44×10^{14} km^4, over the North polar ice - 6.18×10^{14} km^4, and over Himalaya ice - 1.2×10^{14} km^4. The total moment of inertia of thin ellipsoidal shell over the ice is 12.82×10^{14} km^4, and after subtracting of the mean value over the sea ice (5.09×10^{14} km^4, Fig.3) we obtain the moment of inertia of thin ellipsoidal shell over the continental ice $I_{Ic} = 7.73\times10^{14}$ km^4. The ratio between I_{Ic} and I_{OS} ($I_{OS}=109.8\times10^{14}$ km^4) is 0.0704 for ΔM_{SL}=–114m and 0.0019 for ΔM_{SL}=0m. The parabolic dependence between the changes of the inertial moment and ΔM_{SL}, according the curve in Fig.4, is

$$(16)\quad \frac{I_{Ic}}{I_{OS}} = 0{,}0019 + 5\times10^{-5}\Delta M_{SL} + 5.71\times10^{-6}\Delta M_{SL}^2\,.$$

According (9), (15) and (16) the coefficient q varies non-linearly between 0.022 and 0.2 (Fig.5).

Actually, the values of MSL variations in formulae (15) and (16) should represent non-steric MSL variations. The steric sea level variations depend on salinity and temperature changes. Let consider the influence of the sea temperature on the MSL variations during the last glacial maximum. According Hoffert and Covey (1992), the average cooling of oceanic areas not covered with sea ice is 1.75°C during the last glacial maximum. Assuming the termal rate of the water density change -1.2×10^{-4} $g/cm^3/°C$ for the sea depths between 0 and 1000m (with water temperature 10°C - 15°C) and -0.5×10^{-4} $g/cm^3/°C$ for the sea depths between 1000m and 3000m (with water temperature 4°C - 10°C), we determine that the maximal temperature effect on the steric sea level variations is 0.47m during the last glacial maximum. The relative influence of the temperature changes on the steric MSL variations is 0.004 and therefor it is possible to neglect.

Finally the dependence between LOD and glacial MSL variations is (Fig.6)

(17) $L_{OD} = 2.583(1-q)\Delta M_{SL}$,

where MSL variations are in meters and LOD – in milliseconds. According (17) the maximal glacial effects on LOD variations for the last 380Kyr is below 250ms (Fig.6, Fig.7).

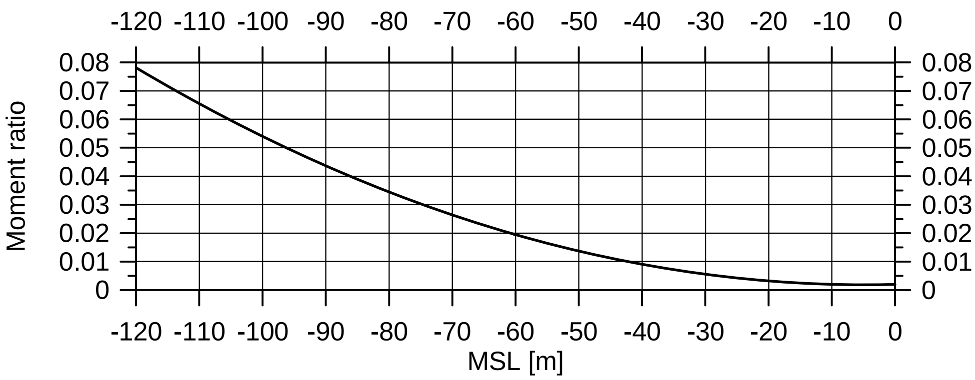

Figure 4. Parabolic dependence between the ratio of I_{Ic} and I_{OS} and changes of the MSL.

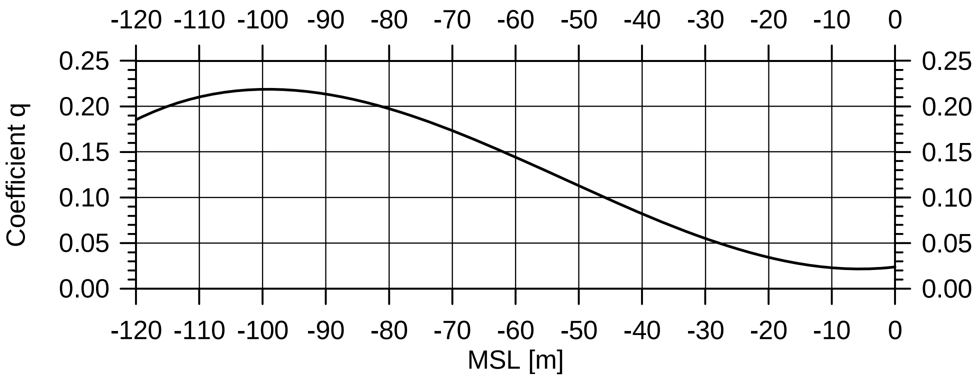

Figure 5. Coefficient *q* of ice sheets influence on the moment of inertia.

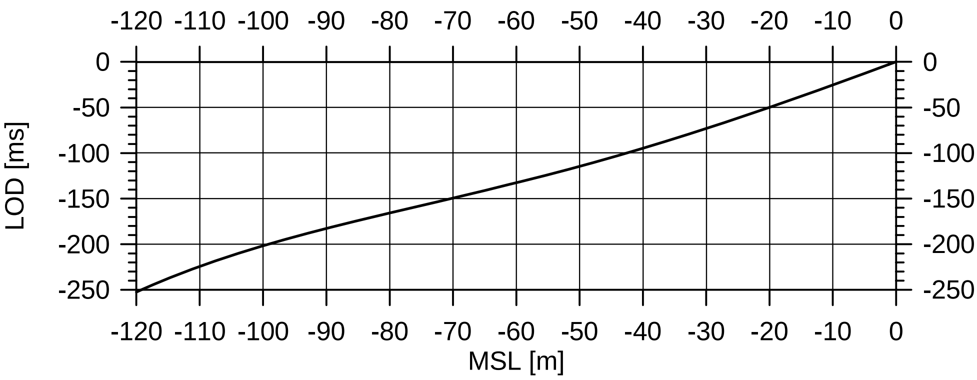

Figure 6. Dependence between LOD and glacial MSL variations.

Periodical components of MSL, LOD and UT1 variations for the last 380Kyr

The glacial LOD variations for the last 380Kyr are determined by means of the MSL data and formula (17). The behavior of the LOD periodicities (Fig.7) is very similar to those of the MSL. The Fast Fourier Transform (FFT) spectra of the LOD and MSL time series (Fig.8) are reveal common frequencies of the long-periodical oscillations, thus the frequencies of LOD response to the glacial MSL variations are not affected by the nonlinear effects of ice sheets influence *q* on the moment of inertia. Both FFT spectrum and Fourier approximation determine significant spectral peaks, corresponding to the oscillations with periods of about 9.7Ka, 10.5Ka, 11.4Ka, 12.8Ka, 14.6Ka, 18.9Ka, 23.5Ka, 39-43Ka, 95-102Ka. The UT1 glacial variations are determined from the LOD time series by numerical integration (Fig.9). The UT1 glacial variations for the last 380Kyr increase almost linear with millennial rate of about 0.548d/Ka. This value does not account the secular decrease of the Earth rotation due to the tidal friction. The glacial change of the Earth angular velocity shows acceleration, which is opposite to the tidal friction effects. The periodical UT1 oscillations are determined by excluding the linear trend from the UT1 time series. The long-periodical UT1 oscillations, due to the glacial MSL variations contain smooth cycles with dominated period of about 100Ka (Fig.10).

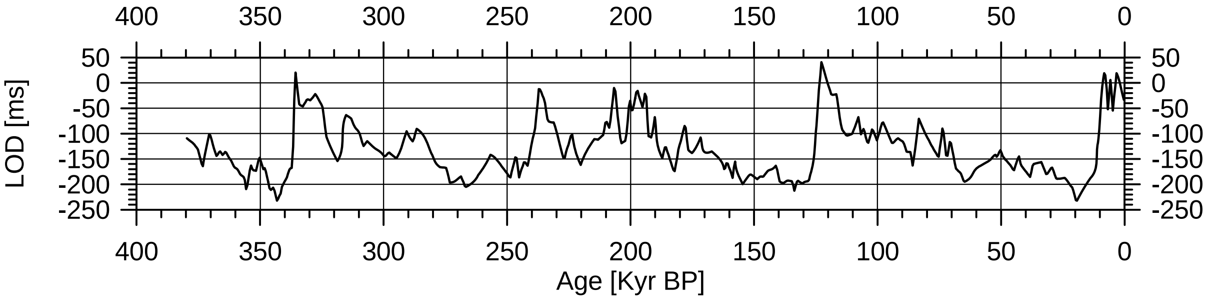

Figure 7. Glacial LOD variations for the last 380Kyr.

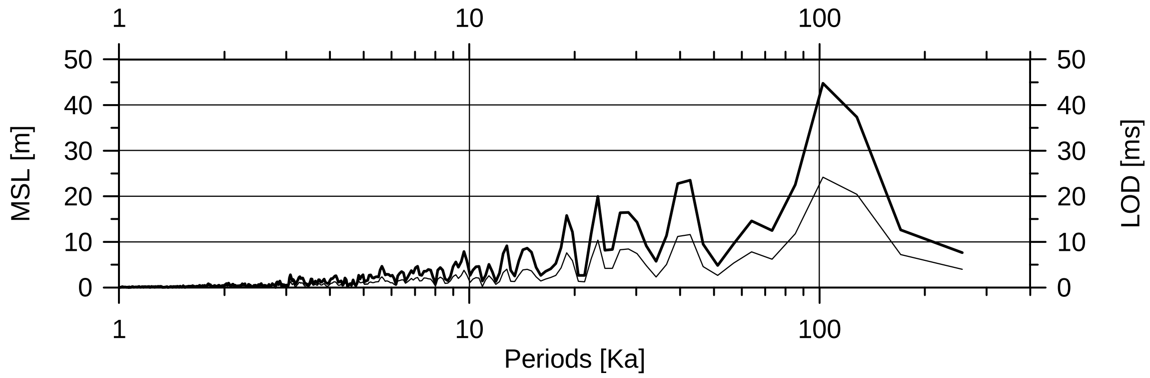

Figure 8. Spectra of LOD (thick line) and MSL (thin line) variations for the last 380Kyr.

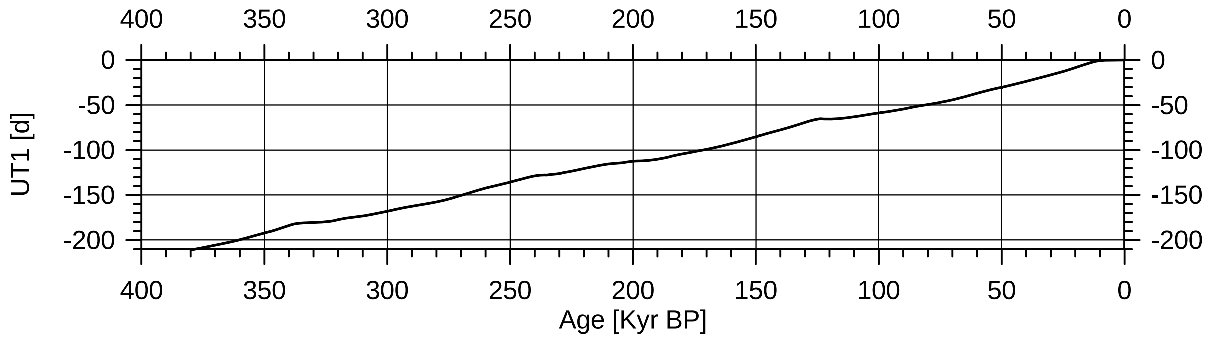

Figure 9. Glacial UT1 changes for the last 380Kyr, determined by numerical integration of LOD.

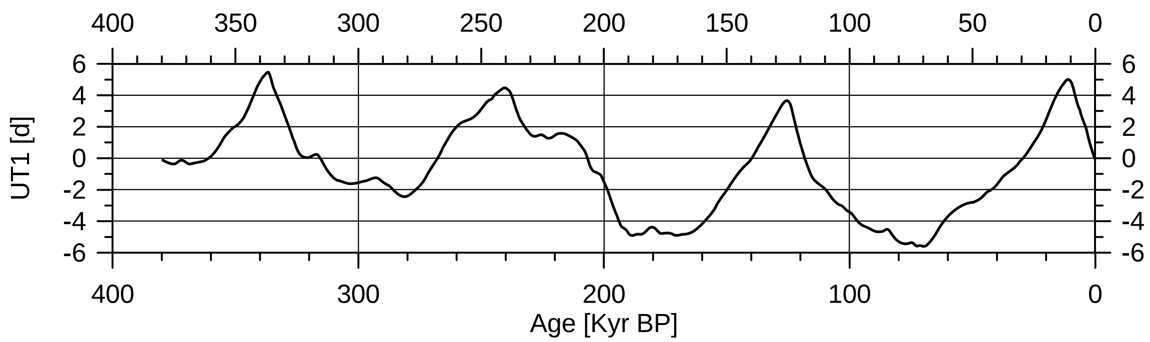

Figure 10. Periodical oscillations of UT1 for the last 380Kyr.

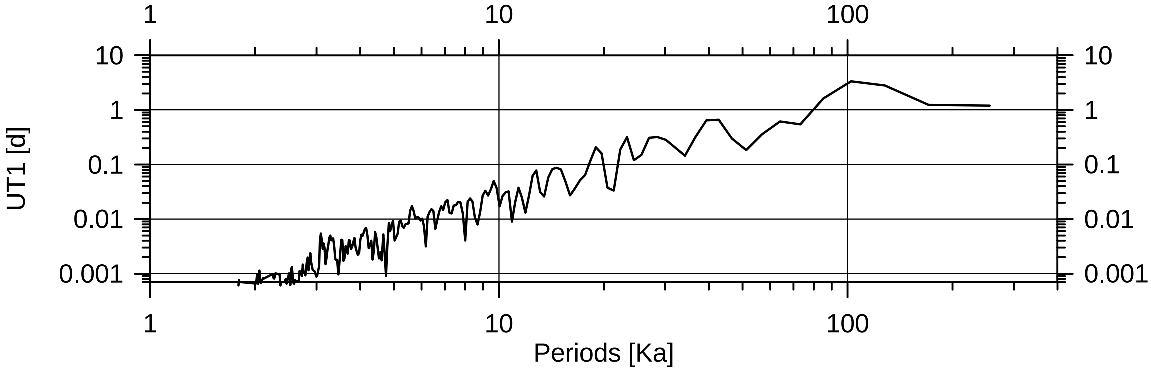

Figure 11. Spectrum of the periodical UT1 oscillations for the last 380Kyr.

The FFT spectrum of the periodical UT1 oscillations for the last 380Kyr contain significant peaks, corresponding to oscillations with periods 9.7Ka, 10.7Ka, 11.4Ka, 12.8Ka, 14.6Ka, 18.9Ka, 23.3Ka, 39-43Ka, 102-128Ka. The periods of the long-term oscillations of LOD and MSL are estimated by means of the iterative regression method (ARIST), which is an iterative least square method for determination the amplitudes, phases and periods of unknown oscillations in the observational and time series. It is described by Wolberg (1967), Rigozo and Nordemann (1998) and Rigozo et al. (2005). This method determines a few of the UT1 oscillations only and a lot of LOD and MSL oscillations. The UT1 oscillations, detected by ARIST method are with periods 24.7Ka, 64.1Ka, 83.7Ka, 100.3Ka and 142.2Ka.

Table 1. Periods and amplitudes of MSL and LOD oscillations, determined by ARIST method.

MSL		LOD	
Period [Ka]	Amplitude [m]	Period [Ka]	Amplitude [ms]
221.0	1.9	237.9	2.5
100.7	16.6	123.4	30.2
72.0	4.8	71.2	7.4
41.1	12.6	45.3	3.8
39.7	11.8	42.5	18.0
38.4	7.7	41.6	23.0
30.0	2.5	32.4	11.0
28.1	4.4	29.5	6.9
		27.0	12.1
26.9	5.8	26.3	2.7
23.7	4.0	24.7	4.8
23.4	4.5	23.8	8.1
23.2	3.9		
22.3	1.1		
18.6	3.1	19.9	4.0
		18.9	13.6
		18.8	11.2
16.5	3.1	16.6	5.5
16.2	3.7	16.2	8.3
14.1	2.5	14.7	4.1
		14.3	7.2
		14.0	3.2
12.8	2.1	12.8	4.0
9.7	2.8	10.7	2.2
		9.7	5.8
		9.6	4.3

Parts of MSL and LOD oscillations with significant amplitudes, estimated by ARIST method, are shown in Table 1. The presence of multiple oscillations with closed frequencies around the spectral peaks means that the intrinsic oscillations are with variable in time amplitudes and phases. The oscillations with periods 18Ka-23Ka are connected with the changes of high North latitude insolation, due to precession variations of the Earth rotation axis. The oscillations with periods around 40Ka and 100Ka are connected with the variations of the eccentricity of the Earth orbit around the Sun (so-called Milankovitch cycles). The correct comparison between Earth orbital parameters variations and MSL, LOD and UT1 cycles need to involve more glacial data to resolve the discrepancies between the periods of MSL and LOD oscillations in Table 1 and to separate the oscillations with orbital origin from the oscillations with other source of excitation.

Conclusions

The reconstructed glacial variations of the sea level for the last 380Kyr give good opportunity for studying the millennial scale variations of the Earth rotation and to determine long-periodical oscillations of the Length of Day LOD and the Universal Time UT1. The method determines the changes of the Earth axial moment of inertia due to global water redistribution between the ocean and continental ice, which is connected with the sea level variations. Two dimensionless coefficients play a key role for proper determination of the glacial variations of the Earth rotation. The first is transfer coefficient k between the homogeneous elastic sphere and the real Earth. The physical means of this coefficient is the correspondence between uniform changes of the radius of an ideal homogeneous elastic sphere and mean sea level of the real Earth ocean with equal changes of the axial moment of inertia. This coefficient is equal to 10.5 in the case of significant changes of the MSL. The other coefficient q represents the relative opposite effects of the ice sheets on the moment of inertia. This coefficient is variable with MSL changes and its maximal value is 0.22 during the last glacial period.

The LOD variations for the last 380Kyr are with maximal deviation -230ms during the glacial maxima. The long-periodical LOD oscillations are with amplitudes from several milliseconds to 45ms and frequencies closed to some frequencies of the Earth orbital variations. The Universal Time UT1 glacial variations for this period consist of linear trend with millennial rate of about 0.548d/Ka. This trend is opposite to the secular decrease of the Earth rotation due to the tidal friction. The periodical part of the UT1 glacial variations consists of significant 100Ka non-sinusoidal cycles with amplitude of about 3.3d and shorter cycles with amplitudes between 0.01d and 1d. Most of these oscillations are with frequencies similar to the frequencies of the Earth orbital variations.

A part of the millennial scale oscillations of the MSL and Earth rotation are outside the frequency band of the Earth orbital variations. The detailed study of the origin of these oscillations need to involve more glacial data of the temperature, isotopes, aerosols, dust, pollens and etc.

Acknowledgements

The present was supported by contract DO 02-275 with the Bulgarian NSF.

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